

# Optimal Solution Search for the Origami Checkerboard Puzzle

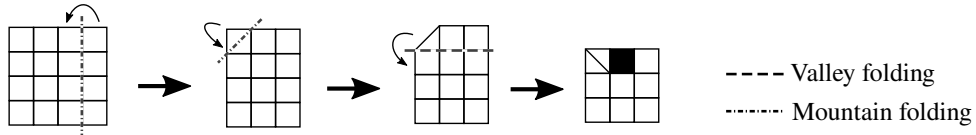
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## Abstract

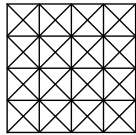
Folding a checkerboard is well investigated topic in computational origami (see, e.g., (DDKL2009)). In the puzzle society, since finding the smallest number of foldings of a specific pattern was proposed as a puzzle by Grabarchuk in 1990s, especially in the Japanese puzzle society, it is well investigated for each of 50 possible checkered patterns of size  $3 \times 3$ . The current records can be found on a website<sup>1</sup>, and these records have been still improved even in 2017. That is, these solutions give upper bounds, and we have no guarantee that they are optimal. We aim at finding optimal steps of folding for these checkered patterns by a supercomputer.

This puzzle starts from a sheet of square paper, and it allows to simple fold. The optimality is defined by the number of simple foldings, and a larger square is better when the steps are tie. We show an example of the pattern #02 and its current record in Figure 1.

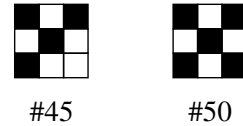


**Figure 1:** Known folding steps of pattern #02.

The number of the longest steps in the current records is 6, and the grid size of the original square is  $9 \times 9$ . Thus, we search up to 6 steps with grid size from  $4 \times 4$  to  $9 \times 9$ .



**Figure 2:** Possible folding lines on a  $4 \times 4$  grid.



**Figure 3:** The pattern #45 and #50.

In this puzzle, we have no restriction for directions of folding, however, it is unlikely that we will use other crease lines than orthogonal or diagonal of  $45^\circ$  lines. Therefore, as shown in

<sup>1</sup><http://puzzlewillbeplayed.com/Origami/OrigamiCheckerboard/>

Figure 2, we only use crease lines along the grid lines and  $45^\circ$  diagonal lines joining the grid points.

Each folding step consists of five parameters; the direction of the crease line, position of the crease line, mountain or valley folds, which side is folded along the line, and how many folded layers. With these parameters, we can fold a set of the isosceles right triangles in a computer.

We use breadth first search on a supercomputer (SGI UV3000; 71.27TFLOPS with 32TB memories). We use trie (keyword tree) to reduce duplicates with checking rotations, reverse, and reflection. The current results are shown in Table 1.

**Table 1:** Each  $\circ$  indicates checked pattern.

steps	grid size					
	$4 \times 4$	$5 \times 5$	$6 \times 6$	$7 \times 7$	$8 \times 8$	$9 \times 9$
1 ~ 4	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$
5	$\circ$	$\circ$	$\circ$			
6	$\circ$					

In the results, we confirmed that the known records are optimal and we cannot improve any more except the pattern #45 and #50 (Figure 3). Moreover, we have found several new ways of folding that achieve tie records. For both of the pattern #45 and #50, the current record is done by 6 folding steps using the square of paper of size  $9 \times 9$ . From the viewpoint of the enumeration, we have enumerated all distinct folded states, except ones whose width or height is a single grid size. We pruned these thin shapes since we will not have any  $3 \times 3$  pattern from them. The summary of the enumeration is shown in Table 2. Blank spaces in the table are being calculated.

**Table 2:** Enumerated distinct folded states.

steps	grid size					
	$4 \times 4$	$5 \times 5$	$6 \times 6$	$7 \times 7$	$8 \times 8$	$9 \times 9$
1	6	7	9	10	12	13
2	223	352	552	746	1,027	1,287
3	7,799	16,835	32,808	54,027	863,17	124,828
4	224,725	684,711	1,656,722	3,369,637	6,231,070	10,495,802
5	4,184,072	19,531,408	60,573,020			

## References

- [DDKL2009] E. D. Demaine, M. L. Demaine, G. Konjevod, and R. J. Lang, Folding a Better Checkerboard, *Annual International Symposium on Algorithms and Computation (ISAAC 2009)*, LNCS Vol. 5878, pp. 1074-1083, 2009.